5. Self & Mutual Induction

Self inductance :- Introduction :-

A wire or Conductor of certain length when twisted into coil becomes a basic inductor. for every conductor carrying Current I and producing Magnetic field $\bar{B}$ there exists a self inductance.

when two such coils are placed very close to each other there exists a Mutual inductance b/n the two coils.

self Inductance :-

when a closed conducting path or a circuit carries current I a Magnetic field $\bar{B}$ is produced. This causes a Magnetic flux ϕ which is given by

$$\phi = \int \bar{B} \cdot ds$$

In this circuit consists of no. of turns n the flux produced by the Magnetic field $\bar{B}$. The flux linkage is defined as λ. The product of No. of Turns & total flux (ϕ)

$$\boxed{\lambda = N\phi} \text{ wb-Turn}$$

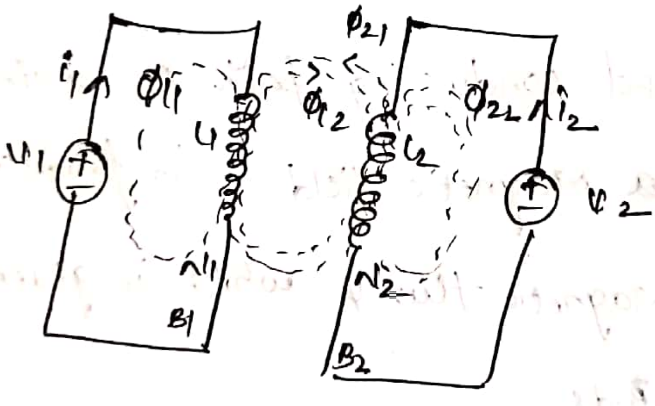
the total ratio of flux linkage to the current flowing through the circuit is called inductance (L). it is given by

$$\boxed{L = \frac{N\phi}{I}\ H = \frac{\lambda}{I}\ H}$$

This inductance is also called as self-inductance.

Mutual inductance:-

consider 2-different circuits with self-inductances L_1 & L_2 are kept close to each other as shown in figure.



Let N_1 & N_2 be the No of turns for the 2 circuits & I_1 & I_2 be the currents through 2 circuits as shown in figure. As 2 circuits are placed very closed to each other these circuits interact magnetically with each other. The flux produced by circuit 1 due to current I_1 flowing through it. it is denoted by ϕ_{11}

Similarly the flux produced by ckt 2 due to current I_2 flowing through it and it is denoted by ϕ_{22}.

The flux produced by each circuit-1 links with ckt-2 and it is denoted by ϕ_{12}.

similarly the flux produced by ckt-2 links with ckt-1 denoted by ϕ_{21}.

the Mutual inductance b/n 2 circuits is defined as the flux linkage of one circuit-1 to current in other ckt.

thus, the inductance M_{12} is given by,

$$M_{12} = \frac{\lambda_1}{I_2} = \frac{\text{flux linkage of ckt-1}}{\text{current in ckt-2}}$$

$$\therefore M_{12} = \frac{\lambda_1}{I_2} = \frac{N_1 \phi_{12}}{I_2} \; H \longrightarrow (1)$$

Similarly

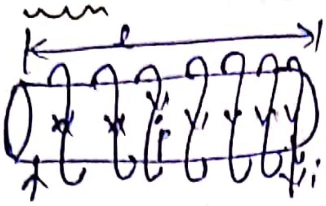
$$M_{21} = \frac{\lambda_2}{I_1} = \frac{N_2 \phi_{21}}{I_1} \; H \longrightarrow (2)$$

If 2 ckt-s are linear then the Mutual inductance represented in eqn (1) & (2) are equal.

$$\therefore \boxed{M_{12} = M_{21} = M}$$

the mutual inductance is also Measured in H.

Inductance of Solenoid:-



solenoid:- A solenoid consists of long conducting wire made up of many loops packed closely to-gether is called as solenoid.

Derivation:-

consider a solenoid of N turns as shown in fig:
let the Current flowing through the solenoid be I amps
let the length of the solenoid be l mt and the cross sectional Area be A

from the results obtained in previous Sections
The Magnetic field intensity due to solenoid is,

$$H = \frac{NI}{l} \ A\text{-}T/mt \longrightarrow (1)$$

The total flux linkage is given by

$$\lambda = N\phi$$

$$= N(BA)$$

$$= N(\mu H)A$$

$$\lambda = N\mu\left[\frac{NI}{l}\right)A$$

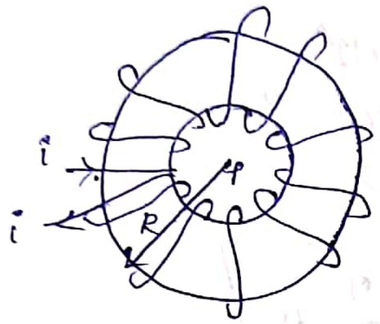
$$\lambda = \frac{\mu N^2 I A}{l} \rightarrow (2)$$

WKT $L = \frac{\lambda}{I}$

$$= \frac{\mu N^2 \cancel{I} A / l}{\cancel{I}}$$

$$\boxed{L = \frac{\mu N^2 A}{l} \ H}$$

Inductance of a toroid:-



toroidal ring

Toroid:- If a long solenoid is bent in the form of a ring and there by closed on itself it becomes a toroid/ solenoidal toroid.

Derivation:-

consider a toroidal ring with N turns and carrying current I. let the radius of the toroid be R as shown in fig: from previous sections.

from the results obtained in previous

The magnetic field intensity is given by

$$H = \frac{NI}{l}$$

$$B = \mu H$$

$$= \mu \frac{NI}{l}$$

$$B = \frac{\mu N I}{2\pi R} \qquad [l = 2\pi R]$$

$$\lambda = N\phi$$

$$= N(BA)$$

$$= N\frac{\mu N I}{2\pi R} \cdot A$$

The self inductance of a toroid is given by $L = \frac{\lambda}{I}$

$$L = \frac{\frac{N\mu N \cancel{I}}{2\pi R} \cdot A}{\cancel{I}}$$

$$\boxed{L = \frac{\mu N^2 A}{2\pi R} \ H}$$

* calculate the inductance of a solenoid of 200 turns wound tightly on a solenoid tube of 6cm diameter. The length of the tube is 60cm and the solenoid is in air.

Sol.

$$L = \frac{\mu N^2 A}{l}$$

$\mu_r = 1$ [it is in air]

$$= \frac{4\pi \times 10^{-7} \times 200 \times 200 \times \pi \times (3 \times 10^{-2})^2}{60 \times 10^{-2}}$$

$$L = 2.36 \times 10^{-4} \ H$$

radius having $\mu_r = 100$ and carrying 900 turns of wire.

Sol:
$\mu_r = 100$

$$L = \frac{N^2 \mu A}{l} = \frac{\mu_0 \mu_r A N^2}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 100 \times \pi \times (2\times10^{-2})^2 \times 900 \times 900}{8\times10^{-2}}$$

$$= \cancel{1.2\times10} \; 1.598 H$$

* A coil of 500 turns is wound on a closed Iron ring of mean radius 10cm and cross sectional area of $3 cm^2$. find self inductance of the winding if the relative permiability of iron is 800.

Sol:
$$L = \frac{\mu N^2 A}{2\pi R}$$

$$= \frac{\mu_0 \mu_r N^2 A}{2\pi R}$$

$$= \frac{4\pi \times 10^{-7} \times 800 \times 500 \times 500 \times 3\times10^{-4}}{2\pi \times 10\times10^{-2}}$$

$$= \cancel{0.0463} \; 0.12 \; H$$

* derive the ~~formula~~ formula for Self inductance of a solenoid using this formula find self inductance of a solenoid having 500 turns Mean diameter is 10 cm and length = 5 cm. Assume Medium to be air.

Soln:-

$$L = \frac{\mu N^2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 500 \times 500 \times \pi \times (5 \times 10^{-2})^2}{5 \times 10^{-2}}$$

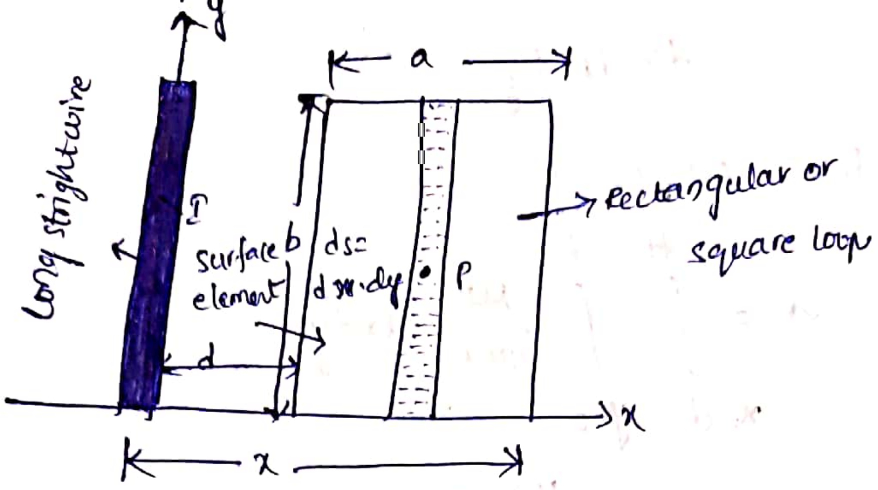
$$= 0.049 H$$

Note:- for a toroid with No. of turns N and the height of the toroid is 'h' with R_1 as inner radius and R_2 as outer radius then the inductance of toroid is given by

$$\boxed{L = \frac{\mu N^2 h}{2\pi} \ln\left[\frac{r_2}{r_1}\right] H}$$

$$- M = \frac{\mu N_1 N_2 A}{l} H$$

* Mutual inductance b/n long straight wire & Rectangular loop in the same plane



Description:-

let the straight long conductor carrying a current I as shown in fig: . the square loop also shown in the same plane. The flux density at any point p in the xy plane at a distance x from the wire is given by expression for amphere's circuit law is,

$$\oint H \cdot dl = I$$

$$H \oint dl = I$$

$$H(2\pi x) = I \qquad l = 2\pi R$$

$$\boxed{H = \frac{I}{2\pi x}}$$

WKT $B = \mu H$

$$\boxed{B = \mu \frac{I}{2\pi x}} \longrightarrow (1)$$

W.K.T The flux linkage,

$$\lambda = \psi = \int_S B \cdot ds \longrightarrow (2)$$

from the figure the surface element is

$$ds = dx \cdot dy \rightarrow (3)$$

substitute (1) & (3) in (2)

$$\lambda = \int_{x=d}^{d+a} \int_{y=0}^{b} \frac{\mu I}{2\pi x} \, dx\,dy$$

$$= \frac{\mu I}{2\pi} \int_{x=d}^{d+a} \int_{y=0}^{b} \frac{1}{x} \, dx\,dy$$

$$= \frac{\mu I}{2\pi} \int_{x=d}^{d+a} \frac{1}{x} \cdot [b-0] \, dx$$

$$= \frac{\mu I b}{2\pi} \int_{x=d}^{d+a} \frac{1}{x} \, dx$$

$$= \frac{\mu I b}{2\pi} \log(x) \Big|_{d}^{d+a}$$

$$= \frac{\mu I b}{2\pi} \log(d+a) - \log d$$

$$= \frac{\mu I b}{2\pi} \log\left[\frac{d+a}{d}\right]$$

WKT $M = \lambda/I$

$$= \frac{\frac{\mu_0 I b}{2\pi} \log\left[\frac{d+a}{d}\right]}{I}$$

$$\boxed{M = \frac{\mu b}{2\pi} \log\left[\frac{d+a}{d}\right]}$$

* **Energy stored in a Magnetic field:-**

- In order to establish a Magnetic field around a coil, energy is required. This Energy stored in a Magnetic if the current is increased from 0 to I with the potential difference across the inductor equal to v. then the source is supplying power equal to vI.

- Energy supplied by the source in time dt is $vI \times dt$ then the energy supplied must be stored in the inductor.

- let dw be the workdone to increase the current by dI. by the law of conservation of energy workdone is equal to energy stored.

$$dw = vI\,dt$$

$$dw = I\left[L\frac{di}{dt}\right]dt$$

$$w = \int_0^I I\,L\,di$$

$$w = \frac{LI^2}{2}$$

$$\boxed{w = \frac{1}{2}LI^2} \text{ Joules} \rightarrow (1)$$

wkT $L = \lambda / I$

$I = \lambda / L \longrightarrow (2)$

from (1) & (2)

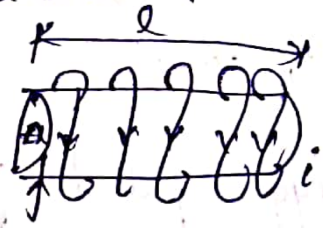
$w = \frac{1}{2} L \left[\frac{\lambda^2}{L^2} \right]$

$$\boxed{w = \frac{1}{2} \frac{\lambda^2}{L} \text{ Joules}} \longrightarrow (3)$$

Energy density in a Magnetic field:-

Energy stored per unit volume is called as Energy density $w_d = \frac{\text{Energy stored}}{\text{volume}}$

consider a coil of N turns wound over a long solenoid of length l mt and uniform cross-sectional area A m^2.



Inductance of solenoid $L = \frac{\mu N^2 A}{l} \longrightarrow (1)$

wkT Energy stored $w = \frac{1}{2} L I^2 \longrightarrow (2)$

substitue (1) in (2)

$w = \frac{1}{2} \frac{\mu N^2 A}{l} \cdot I^2$

$= \frac{1}{2} \frac{\mu N^2 A \cdot l I^2}{l^2}$

$= \frac{1}{2} \mu \left[\frac{NI}{l} \right]^2 l A$

$$= \frac{1}{2}\mu H^2 (lA)$$

$$W = \frac{1}{2}\mu H^2 \times \text{volume}$$

$$\frac{\text{energy stored}}{\text{volume}} = \frac{1}{2}\mu H^2$$

$$\boxed{W_d = \frac{1}{2}\mu H^2 \; J}$$

$$\boxed{W_d = \frac{1}{2} BH \; J}$$